TRANSFER PROCESSES WHEN THE SURFACE MATERIAL OF A BODY AND AN INJECTED MATERIAL REACT WITH A LAMINAR BOUNDARY LAYER

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The article constitutes an analytical investigation of an unsteady (due to a change in the boundary of the porous body) laminar boundary layer. There is a chemical reaction (for infinitely large rates of heterogeneous and homogeneous reactions) between the wall surface material together with the injected material and a coolant contained in the external flow. For the particular case $\mathrm{Pr} = \mathrm{Sc}_i = 1$, a solution in closed form is obtained for problem (31)–(32).

When applying boundary layer theory to laminar flows with chemical reactions it is usual to take into account either the reaction of an oxidizer being filtered through a chemically inert porous wall with the gas of the external stream [4-6], or breakdown of the material of the body surface (sublimation, combustion, etc.) in the absence of insufflation [2].

In the present paper both phenomena are taken into account, i.e., an analytical investigation is performed for cases in which a coolant is blown through a porous wall or the coolant and the wall react with the laminar boundary layer. Similar solutions have been obtained for the case of blowing of hydrogen through a porous graphite wall washed by a uniform stream of air, chemical reaction of the graphite and hydrogen with the oxygen of the external stream, i.e., combustion, occurring at the body surface.* Here it is assumed that the high temperature conditions will effect equilibrium for the heterogeneous reaction

$$2C + O_2 = 2CO \qquad \text{(a)}$$

and for the homogeneous reaction

$$2H_2 + O_2 = 2H_2O \qquad \text{(b)}$$

occurring in an infinitely thin zone at the body surface, considered as a separation surface. As a result of interaction of the hydrogen and the graphite of the wall with oxygen of the air at the body surface, only water and carbon dioxide will be formed, respectively [3], these reactions proceeding both in the boundary layer and within the wall (in a mixture with an inert component, i.e., nitrogen). We cannot consider the equations of internal mass transfer, since the weight fraction of the gases entering the wall decreases rapidly, and has no influence on the concentration distribution in the boundary layer. The rate of inflow of oxygen corresponds to the stoichiometric value required for full oxidation of hydrogen and carbon, as a result of which the concentrations of H_2 and O_2 in the reaction zone tend to zero. The volume porosity P of the wall is equal to the surface value, and we may neglect the thermodiffusion effect. In addition, close to the wall surface, i.e., in the high temperature zone, the reaction of H_2 with C, forming C_2H_2 (or C_2H_4) is possible. These gases become oxidized, as was true in reactions (a) and (b), to give CO and H_2O. Consequently, the intermediate reaction of H_2 with C (whose rate $W^- \to 0$, since $C_C \to 0$ as a result of reaction (b)) has no influence on the thermal effect, and need not be considered.

In accordance with the process scheme being considered, there is no hydrogen in molecular form in the compressible gas stream, and there is no O_2 component within the porous wall. Therefore, the following gases will be found within the volume of the boundary layer: O_2, H_2O, CO and N_2. It is clear that these components may be divided into three distinct groups as regards molecular weight, and therefore we have to consider multi-component diffusion for an accurate description of the process.

If we consider a gas mixture consisting of N components (for the conditions examined N = 4), this problem reduces to solution of a system of equations for an unsteady reacting boundary layer

$$\frac{\partial \rho}{\partial \tau} + \operatorname{div}(\rho \mathbf{v}) = 0, \qquad (1)$$

$$\rho \frac{du}{d\tau} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \qquad (2)$$

$$\rho \frac{dC_i}{d\tau} = \frac{\partial}{\partial y}\left(\rho D_i \frac{\partial C_i}{\partial y}\right), \quad i = 1, 2, 3, \qquad (3)$$

$$\rho \frac{dH}{d\tau} = \frac{\partial}{\partial y}\left[\frac{\mu}{\mathrm{Pr}}\left(\frac{\partial H}{\partial y}\right) + \mu\left(1 - \frac{1}{\mathrm{Pr}}\right)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) + \sum_{i=1}^{N=4} \rho D_i \left(1 - \frac{1}{\mathrm{Le}_i}\right) h_i \frac{\partial C_i}{\partial y}\right], \qquad (4)$$

$$\sum_{i=1}^{N=4} C_i = 1, \qquad (5)$$

supplemented by an equation of state

$$p = R\rho T \sum_{i=1}^{N=4} \frac{C_i}{m_i}, \qquad (6)$$

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*Laminar flow of a liquid and gas in a nonreacting boundary layer with strong surface separation has been investigated in detail [1,2]. The present paper examines conditions at the separation surface coinciding with the boundary of the body, in the presence of chemical reactions.

and the equation for heat conduction in the porous wall

$$\rho_\Sigma c_{P\Sigma}\frac{\partial T_\Sigma}{\partial \tau}=\frac{\partial}{\partial y}\left(\lambda_\Sigma\frac{\partial T_\Sigma}{\partial y}\right)-c_{P\Gamma}(\rho v)_\Gamma\frac{\partial T_\Sigma}{\partial y}. \tag{7}$$

Here C_i ($C_i = \rho_i/\rho$) is the weight concentration of the i-th gas component; H ($H = h + u^2/2$) is the total, and $h\left(h=\sum_{i=1}^{N}h_i^* C_i\right)$ the static enthalpy; $h_i^*\left(h_i^*=\int_{T_0}^{T}c_{Pi}dT+h_i\right)$ is the enthalpy of component i; h_i is the enthalpy of formation of the component at temperature T_0; ρ_Σ, $c^*_{P\Sigma}$ ($c^*_{P\Sigma} = c_{P\Sigma}\rho_\Sigma$) and λ_Σ are, respectively, the effective density, volume heat capacity at constant pressure, and thermal conductivity of the porous wall, as determined from the relation

$$\xi_\Sigma=(1-P)\xi_C+P\xi_H, \tag{8}$$

where $\xi \equiv c_P^*$, ρ, and λ.

In writing (7), it was assumed that the characteristic length of the test body is large enough in comparison with the thickness of the thermal boundary layer in the porous wall, i.e., $\partial/\partial x \ll \partial/\partial y$.

Boundary Conditions at the Wall. Taking into account that the C and H components of the surface take part in the heterogeneous and homogeneous reactions, we shall find the mean mass fluxes J_i (i = 1, 2, 3, 4, H, C) being formed, and also the dependence between J_i and the stoichiometric coefficients r_K:

for reaction (a)

$$r=-\frac{2m_C}{m_1}=\frac{J_C}{J_1^C},\quad r_{II}=\frac{m_C}{m_2}=\frac{J_C}{J_2}, \tag{9}$$

for reaction (b)

$$r_I=-\frac{2m_H}{m_1}=\frac{J_H}{J_1^H},\quad r_{III}=\frac{m_H}{m_3}=\frac{J_H}{J_3}. \tag{10}$$

(Subscripts 1, 2, 3, 4, H, and C denote, respectively, components O_2, CO, H_2O, N_2, H_2 and C, $J_1 = J_1^C + J_1^H$).

We shall designate $r = z^{-1}$. Then, from the law of conservation of mass in reactions (a) and (b), we must have $z + z_{II} = 1$, $z_I + z_{III} = 1$.

In the general case the flux J_i may be represented in the form

$$J_i=\rho(v-V)C_{in}+j_i \quad (i=1,\ 2,\ 3,\ 4), \tag{11}$$

where V is the velocity of displacement of the body surface and j_i is the diffusion flux ($j_i = -\rho D_i \cdot \partial C_i/\partial y$). It should be noted that in the given coordinate system, the vectors $V < 0$ and $j_1 < 0$, and the directions of the fluxes j_1 and j_i (i = 2, 3, 4) are opposite. Also $J_1 = j_1$, since $C_{1s} = 0$. Summing equations (14) over the subscript i (i = 1, 2, 3, 4), we obtain the condition for conservation of mass of the mixture at the separation surface

$$\rho(v-V)=J_\Sigma, \tag{12}$$

where J_Σ is the total flow rate of the material ablating from the porous wall and of coolant filtering through it. Since $(\rho_H v_H)_e = (\rho v)_s^-$ and $\rho_\Sigma = P\rho_s^- + \rho_C(1 - P)$, we have

$$J_\Sigma=P\rho_s^-\left(\frac{v_s^-}{P}-V\right)-(1-P)V\rho_C=$$
$$=(\rho_H v_H)_e-\rho_\Sigma V. \tag{13}$$

When P = 0, $J_\Sigma = -V\rho_S$, which corresponds to the case of ablation of a solid wall, for example when it sublimates [2] or burns. If P = 1, $J_\Sigma = \rho_s^-(v_s^- - V)$, and (12) transforms to the ordinary form [1].

It is clear that

$$J_C=-(1-P)V\rho_C. \tag{14}$$

Since $C_H = 0$ and $J_H = j_H^-$ at the wall in laminar flow, we have therefore the following analytical expression for determining the mean flow rate of hydrogen from a unit surface of the body:

$$j_H^-=-\left(\rho D_H\frac{\partial C_H}{\partial y}\right)_w^-.$$

However, since the distribution of the concentrations of gases within the porous wall is unknown, we determine the numerical value of j_H^- to be

$$j_H^-=P\rho_{He}\left(\frac{v_{He}}{P}-V\right)=(\rho_H v_H)_e-PV\rho_{He}. \tag{15}$$

Using the relations (9)–(15), we can write down the boundary conditions at the body surface:

$$\rho(v-V)=(\rho_H v_H)_e-\rho_\Sigma V; \tag{16}$$

$$u=0,\quad C_1=0,\quad C_i=C_{is}\quad(i=\ 2,\ 3,\ 4),$$
$$T=T_\Sigma=T_s; \tag{17}$$

$$-j_1=r^{-1}J_C+r_I^{-1}J_H,\quad r_{II}[\rho(v-V)C_{2s}+j_2]=J_C,$$
$$r_{III}[\rho(v-V)C_{3s}+j_3]=J_H; \tag{18}$$

$$\rho(v-V)Q(T_s)=\lambda_\Sigma\frac{\partial T_\Sigma}{\partial y}-\lambda\frac{\partial T}{\partial y}. \tag{19}$$

Here $Q(T_s)$ is the heat of reaction during burning of hydrogen and of the graphite wall:

$$Q(T_s)=Q_{RC}J_C/J_\Sigma+Q_{RH}J_H/J_\Sigma,$$

where $J_\Sigma = J_C + J_H$; Q_{RC} and Q_{RH} are, respectively, the heat of reaction per unit mass of carbon and hydrogen. Bearing in mind reactions (a) and (b), we find

$$Q_{RC}=h_C+\frac{m_1}{2m_C}h_1-\frac{m_2}{m_C}h_2,$$

$$Q_{RH}=h_H+\frac{m_1}{2m_H}h_1-\frac{m_3}{m_H}h_3.$$

In determining the density ρ_s^- of the gas mixture at the body surface in the porous wall, which enters as a factor in (8), (13), and (14), it should be remembered that $C_{Hs} = C_{1s} = 0$, $C_{He} = 1$. Then, from the equation of state (6), taking into account the last inequalities, we have, for the mixture of gases inside a porous wall,

$$p_e = \frac{R}{m_{\mathrm{H}}}(\rho_{\mathrm{H}} C_{\mathrm{H}} T),$$

$$p_{\mathrm{s}}^- = \rho_{\mathrm{s}}^- T_{\mathrm{s}} \sum_i \frac{C_i}{m_i} \quad (i = 2,\ 3,\ 4).$$

Since $v_{\mathrm{s}}^- = (\rho_{\mathrm{He}}/\rho_{\mathrm{s}}^-) v_{\mathrm{He}}$ and $p_e = p_{\mathrm{s}}^-$, therefore,

$$\rho_{\mathrm{s}}^- = \rho_{\mathrm{He}} \left(\frac{T_{\mathrm{s}}}{T_e} m_{\mathrm{H}} \sum_i \frac{C_{i\mathrm{s}}}{m_i} \right)^{-1} \quad (i = 2,\ 3,\ 4).$$

The first term in (16) obtained by summation in (9) and (10) of the relations with coefficient r and r_{I}, is the stoichiometric relation between the flow rates of components of the porous wall (carbon J_{C} and the filtered hydrogen J_{H}), necessary for full oxidation of these by the flux j_1 of oxygen.

The boundary conditions in the stream at infinity are

$$u = u_\infty, \quad H = H_\infty,$$

$$C_i = C_{i\infty} \quad (i = 1,\ 4), \quad C_2 = C_3 = 0 \tag{21}$$

and in the porous wall at infinity

$$T_\Sigma = T_e\,. \tag{22}$$

If we consider this kind of ablation regime, when hydrogen is supplied through a flat plate according to

$$v_{\mathrm{He}} = \frac{\rho_\infty}{2\rho_\Sigma} \left(\frac{\nu_\infty u_\infty}{x} \right)^{1/2} B, \tag{23}$$

and the rate of displacement of the surface of separation of the phases, V, is a function only of coordinate x (correspondingly $V \sim x^{-1/2}$), and does not depend on time,

$$V = \frac{\rho_\infty}{2\rho_\Sigma} \left(\frac{\nu_\infty u_\infty}{x} \right)^{1/2} A \quad (A < 0), \tag{24}$$

then in solving system (1)–(7), with boundary conditions (16)–(19), we may use the following variables [[illegible]]:

$$\tilde{\rho} \tilde{u} = \varphi'_\eta(\eta,\ \tau),$$

$$\tilde{\rho} \tilde{v} = \frac{1}{2} \left(\frac{\nu_\infty}{u_\infty x} \right)^{1/2} \left[\eta \varphi'_\eta - \varphi(\eta,\ \tau) + \frac{\rho}{\rho_\Sigma} A \right],$$

$$T_\Sigma = T_e\, \theta(\eta_\Sigma), \quad \eta = \left(\frac{u_\infty}{\nu_\infty x} \right)^{1/2} (y - V\tau),$$

$$\eta_\Sigma = \frac{V}{\chi_1}(y - V\tau), \quad \chi_1 = \left(\frac{\lambda_\Sigma}{\rho_\Sigma c_{P\Sigma}} \right)_e, \tag{25}$$

where, according to (7), (17) and (22), the function $\theta(\eta_\Sigma)$ is a solution of the boundary problem

$$\frac{d}{d\eta_\Sigma} \left[L(\theta) \frac{d\theta}{d\eta_\Sigma} \right] + N(\theta) \frac{d\theta}{d\eta_\Sigma} = 0,$$

$$\theta(0) = \frac{T_{\mathrm{s}}}{T_e} = n, \quad \theta_e = 1, \tag{26}$$

i.e.,

$$\eta_\Sigma = \int_\theta^n \frac{L(\theta)\, d\theta}{M(\theta) - M(1)}, \quad \frac{d\theta}{d\eta_\Sigma} = \frac{M(1) - M(\theta)}{L(\theta)}. \tag{27}$$

Here

$$N(\theta) = \tilde{N}(\theta) - \psi(\theta) \frac{(\rho_{\mathrm{H}} c_{P\mathrm{H}})_e}{(\rho_\Sigma c_{P\Sigma})_e} \frac{B}{A},$$

$$\rho_{\mathrm{g}} c_{P\mathrm{g}} = (\rho_{\mathrm{g}} c_{P\mathrm{g}})_e\, \psi(\theta) = (\rho_{\mathrm{H}} c_{P\mathrm{H}})_e\, \psi(\theta),$$

$$\lambda_\Sigma = \lambda_{\Sigma e} L(\theta), \quad \rho_\Sigma c_{P\Sigma} = (\rho_\Sigma c_{P\Sigma})_e \tilde{N}(\theta),$$

$$M(\theta) = \int N(\theta)\, d\theta.$$

In order to elucidate the meaning of the dimensionless constants A and B in (23) and (24), we shall find the mass flow rate of carbon, G_{C}, and hydrogen, G_{H}, at the surface of a porous plate of length l:

$$G_{\mathrm{C}} = \int_0^l J_{\mathrm{C}}\, dx = (P - 1) \frac{K_{\mathrm{C}}}{K} \frac{\varkappa}{\sqrt{\mathrm{Re}_\infty}} A,$$

$$G_{\mathrm{H}} = \int_0^l (j_{\mathrm{H}}^-)\, dx = (B - PA) \frac{\varkappa}{\sqrt{\mathrm{Re}_\infty}},$$

where K is the insufflation parameter ($K = \rho_{\mathrm{He}}/\rho_\Sigma$); $K_{\mathrm{C}} = \rho_{\mathrm{C}}/\rho_\Sigma \approx 1$, $\mathrm{Re}_\infty = u_\infty \rho_\infty l/\mu_\infty$, $\varkappa = \rho_\infty u_\infty K l$. Hence

$$A = [(P - 1) K_{\mathrm{C}} \varkappa]^{-1} G_{\mathrm{C}} K \sqrt{\mathrm{Re}_\infty},$$

$$B = G_{\mathrm{H}} \sqrt{\mathrm{Re}_\infty}\, \varkappa^{-1} + PA.$$

Substituting (23)–(25) into (1)–(7) and boundary conditions (16)–(19) we arrive at the following boundary problem:

$$\frac{\partial}{\partial\eta} \left(\tilde{\mu} \frac{\partial \tilde{u}}{\partial\eta} \right) + \frac{\varphi}{2} \frac{\partial \tilde{u}}{\partial\eta} = 0,$$

$$\frac{\partial}{\partial\eta} \left(\frac{\tilde{\mu}}{\mathrm{Sc}_i} \frac{\partial C_i}{\partial\eta} \right) + \frac{\varphi}{2} \frac{\partial C_i}{\partial\eta} = 0$$

$$(i = 1,\ 2,\ 3),$$

$$\frac{\partial}{\partial\eta} \left\{ \frac{\tilde{\mu}}{\mathrm{Pr}} \left[\frac{\partial H}{\partial\eta} + u_\infty^2 (\mathrm{Pr} - 1) \frac{\partial}{\partial\eta} \left(\frac{u^2}{2} \right) + \right. \right.$$

$$\left. \left. + \sum_{i=1}^{N=4} (\mathrm{Le}_i - 1) h_i \frac{\partial C_i}{\partial\eta} \right] \right\} + \frac{\varphi}{2} \frac{\partial H}{\partial\eta} = 0; \tag{28}$$

$$\tilde{u}(\infty) = 1, \quad \tilde{u}(0) = 0,\ H(\infty) = H_\infty,\ C_1(\infty) = C_{1\infty},$$

$$C_i(\infty) = 0 \quad (i = 2,\ 3),$$

$$\varphi(0,\ \tau) = -B^*, \quad 2 \frac{\rho D_1}{\mu_\infty} \frac{\partial C_1}{\partial\eta} = \frac{\bar{J}_{\mathrm{C}}}{r} + \frac{\bar{J}_{\mathrm{H}}}{r_{\mathrm{I}}},$$

$$r_{\mathrm{II}} \left[2 \frac{\rho D_2}{\mu_\infty} \frac{\partial C_2}{\partial\eta} + \varphi(0,\ \tau) C_{2\mathrm{s}} \right] = \bar{J}_{\mathrm{C}},$$

$$r_{\mathrm{III}} \left[2 \frac{\rho D_3}{\mu_\infty} \frac{\partial C_3}{\partial\eta} + \varphi(0,\ \tau) C_{3\mathrm{s}} \right] = \bar{J}_{\mathrm{H}},$$

$$\varphi(0,\ \tau)Q(T_s) = C_{P\Sigma}(T_{s1})T_e A\,\frac{M(1)-M(n)}{N(n)} +$$

$$+\frac{2}{\Pr(T_s)}\left(\frac{\mu_s}{\mu_\infty}\right)\left[\frac{\partial H}{\partial\eta}-\sum_{i=1}^{N=4}h_i\frac{\partial C_i}{\partial\eta}\right], \qquad (29)$$

where

$$\bar{J}_C = (P-1)K_C A,\ \bar{J}_H = K(B-PA),\quad B^* = BK - A.$$

As has been shown in reference [2], the momentum equation in (28) is valid only for sufficiently small initial values of time (of the order of several tens of seconds). In the conditions examined this limitation is extended also to the remaining relations of system (28). The boundary conditions (16)–(19), written in terms of the new variables (23)–(25), are satisfied exactly, and transform to the form (29). In deriving (29) it was assumed that $\rho_{\Sigma e} = \rho_{\Sigma s} = \rho_\Sigma$.

The equations of momentum and energy (28) are of the third order (since $\tilde{\rho}\,\tilde{u} = \varphi_\eta'$) and second order, respectively. Each of the three diffusion equations is of second order. In addition, the two quantities T_s and C_4 are as yet unknown. Relations (29), together with relations (5) and (6), give thirteen conditions.

Considerable mathematical simplifications occur for $\Pr = \mathrm{Sc}_i = 1$ and $\mu\rho = \mathrm{const}$. In this case solutions of the diffusion and energy equations of system (28)–(29) have the form [2]

$$C_i = C_{is} + (C_{i\infty} - C_{is})\,\tilde{u}(\eta)\quad (i = 1,\ 2,\ 3),$$

$$H = H_s + (H_\infty - H_s)\,\tilde{u}(\eta), \qquad (30)$$

and, therefore, if H_s and C_{is} are known, the problem reduces to determining the velocity field $\tilde{u} = \tilde{u}(\eta)$, i.e., to solution of the momentum equation.

The problem (28)–(29) may be simplified by introducing the new desired function $\omega = \tilde{\mu}\partial\tilde{u}/\partial\eta$, and by using $\tilde{u}$ as the independent variable [1]. Then

$$2\omega\omega'' + \tilde{\rho}\tilde{\mu}\,\tilde{u} = 0,$$

$$(\omega\,\bar{j}_i)' + \omega' C_i' = 0\quad (i = 1,\ 2,\ 3), \qquad (31)$$

$$\left\{\omega\left[-\frac{H'}{\Pr}-\left(1-\frac{1}{\Pr}\right)u_\infty^2\,\tilde{u} + \right.\right.$$

$$\left.\left.+\sum_{i=1}^{N=4}h_i\left(\frac{C_i'}{\Pr}+\bar{j}_i\right)\right]\right\}' + \omega' H' = 0;$$

$$\omega(1) = 0,\quad \omega'(0) = 1/2B^*,$$

$$H(1) = H_\infty,\quad C_1(1) = C_{1\infty}.\ C_i(1) = 0\quad (i = 2,\ 3),$$

$$-2\bar{j}_1\,\omega(0) = r^{-1}\bar{J}_C + r_{\mathrm{I}}^{-1}\bar{J}_H,$$

$$2r_{\mathrm{II}}\bar{J}_2 = \bar{J}_C,\quad 2r_{\mathrm{III}}\bar{J}_3 = \bar{J}_H,$$

$$2\omega'(0)Q(T_s) = T_e c_{P\Sigma}(T_s)A\,\frac{M(n)-M(1)}{N(n)} -$$

$$-\frac{2\omega(0)}{\Pr(T_s)}\left(H' - \sum_{i=1}^{N=4}h_i C_i'\right), \qquad (32)$$

where

$$\bar{j}_i = -C_i'/\mathrm{Sc}_i,$$

$$\bar{J}_i = \bar{j}_i\,\omega(0) + \omega'(0)C_{is}\quad (i = 1,\ 2,\ 3).$$

(In (31), (32), and subsequently, derivatives with respect to $\tilde{u}$ are denoted by primes).

The system (31)–(32) must be supplemented by (N − 1) relations giving the fluxes $\bar{j}_i$ as a function of the binary diffusion coefficients D_{iK} and the parameters C_i, C_i', m_i [9]:

$$\frac{\mu}{\rho}\sum_{K=1}^{K=3}\frac{C_K}{m_K D_{iK}}\left(\frac{\bar{j}_i}{C_i}-\frac{\bar{j}_K}{C_K}\right) =$$

$$=\sum_{K=1}^{K=3}\frac{C_K}{m_K}\left(\frac{C_K'}{C_K}-\frac{C_i'}{C_i}\right),\ k \neq i,$$

$$i = 1,\ 2,\ 3,\qquad \sum_{i=1}^{N=4}\bar{j}_i = 0, \qquad (33)$$

and also relations giving the physical coefficients of the gases as a function of temperature and concentration [7]:

$$\mu_i = b(m_i T)^{1/2}\sigma_i^{-2},\quad \lambda_i = c\,\frac{\mu_i}{m_i}R\left(\frac{4}{15}\frac{c_{vi}}{R}+\frac{3}{5}\right),$$

$$D_{iK} = d\,[T^3(m_i + m_K)/2m_i m_K]^{1/2}\,p^{-1}\sigma_{iK}^{-2},$$

$$\mu = \sum_{i=1}^{N=4}\frac{(C_i/m_i)\mu_i}{\sum\limits_{K=1}^{4}\frac{C_K}{m_K}F_{iK}},\quad \lambda = \sum_{i=1}^{N=4}\frac{(C_i/m_i)\lambda_i}{\sum\limits_{K=1}^{4}\frac{C_K}{m_K}F_{iK}},$$

$$c_{Pi} = \bar{b} + \bar{c}T + \bar{d}/T^2,\quad \bar{c}_P = \sum_{i=1}^{N=4}c_{Pi}C_i, \qquad (34)$$

where σ_i is the diameter of the molecule; $\sigma_{iK} = (\sigma_i + \sigma_K)/2$; $F_{ii} = F_{KK} = 1$; $F_{iK} = 1,\ 2(\sigma_{iK}/\sigma_{ii})^2\,[2m_K/(m_K + m_i)]^{1/2}$ for $i \neq K$; c_{vi} is the heat capacity of the i-th component of the gas at constant volume. Values of the coefficients b, c, d and $\bar{b}$, $\bar{c}$, $\bar{d}$ for certain gases are given in reference [5, 7, 9]. For the complex $\tilde{\mu}\tilde{\rho}$ the inequality $\tilde{\mu}\tilde{\rho} = T^{n-1}$ is valid [8]. For high temperatures n = 1/2.

Thus, it is required to solve system (31) of five equations of second order with five unknowns (T_s, C_4 and $\bar{j}_i$, i = 1, 2, 3), with ten conditions (32), and five relations (5), (6), and (33). The flux $\bar{j}_4$ is determined from the second equality of (33), and the thermophysical parameters from (34). Therefore, we have a closed system of equations, boundary conditions, and supplementary conditions, which must be solved by means of numerical integration, although these calculations are quite complicated for the given problem.

We shall restrict ourselves to the case $\Pr = \mathrm{Sc}_i = 1$, $\mu\rho = \mathrm{const}$. Then the solution of the momentum equation, allowing for the boundary condition $\omega(1) = 0$, will be [1]

$$\omega(\tilde{u}) = u_0^{-3/2}\,\omega_\gamma(\tilde{u}u_0),\quad \eta = \int_0^{\tilde{u}}\frac{\tilde{\mu}}{\omega(\tilde{u})}\,d\tilde{u}, \qquad (35)$$

where u_0 are the zeros of the function ω_γ. Tables of $u_0(\gamma)$ and ω_γ are given in [1,2].

From the remaining conditions of (32), taking into account (30) and (35), we obtain a system of five equations

$$u_0^{-1/2} = 1/2B^*,\ C_{1\infty} = 1/2\, u_0^{3/2}(r^{-1}\overline{J}_C + r_I^{-1}\overline{J}_H),$$

$$C_{2S} = \overline{J}_C\left[r_{II}(B^* + 2u_0^{-3/2})\right]^{-1},$$

$$C_{3S} = \overline{J}_H\left[r_{III}(B^* + 2u_0^{-3/2})\right]^{-1},$$

$$B^*Q(T_S) = T_e AC_{p\Sigma}(T_S)\,\frac{M(n) - M(1)}{N(n)} -$$

$$- 2u_0^{-3/2}(H_\infty + h_1C_{1\infty} + h_4C_{4\infty})$$

for calculating the parameters $C_{1\infty}$, C_{2S}, C_{3S}, T_S, and γ ($\mathrm{tg}\,\gamma = \omega'_\gamma(0)$). Since $\gamma = \gamma(u_0)$, then, knowing γ, we can find u_0 from tables. The values $\tilde{u} = \tilde{u}(\eta)$ appearing in (30) are determined by solution of the momentum equation from the second equality of (35). Using the relations

$$c_f = \frac{2\tau_w}{\rho_\infty u_\infty^2},\ \frac{\lambda}{\overline{c}_p}\left(\frac{\partial H}{\partial y} - \sum_{i=1}^{N=4} h_i\frac{\partial C_i}{\partial y}\right)_w = \left(\lambda\frac{\partial T}{\partial y}\right)_w,$$

where $\tau_W = \mu(\partial u/\partial y)_W$, as well as (30), we obtain, after simple transformations, the friction coefficient

$$c_f = 2u_0^{-3/2}/\sqrt{\mathrm{Re}_x}$$

and the heat flux at the wall

$$q_w = u_0^{-3/2}\rho_\infty u_\infty(H_\infty - C_{1\infty}h_1 - C_{4\infty}h_4)/\sqrt{\mathrm{Re}_x}.$$

Thus, for the special case $\mathrm{Pr} = \mathrm{Sc}_i = 1$ and $\mu\rho =$ const the solution of the given problem may be obtained in closed form.

NOTATION

x is the coordinate along the flat plate; y is the coordinate normal to it; u and v are components of flow velocity along x and y, respectively; R is the universal gas constant; p is the pressure; $\mathrm{Pr} = \mu\overline{c}_p/\lambda$, $\mathrm{Sc}_i = \mu/\rho D_i$, $\mathrm{Re} = \rho ul/\mu$, μ is the dynamic viscosity; C_i and m_i are the mass concentration and molecular weight, respectively, of the i-th component; j_i ($j_i = \rho_i g_i = -\rho D_i \cdot \partial C_i/\partial y$) is the diffusion mass flux; $\overline{j}_i$ normalized diffusion mass flux $\overline{j}_i = -\mathrm{Sc}_i^{-1}C'_i$); $\tilde{u} = u/u_\infty$; $\tilde{v} = v/u_\infty$; $\tilde{\rho} = \rho/\rho_\infty$, $\tilde{\mu} = \mu/\mu_\infty$. Subscripts: ∞ stands for conditions at the outer edge of the boundary layer; w means at the wall; x is for the value at the given coordinate; the subscript s refers to values at the body surface, unknown prior to solution of the problem; Σ refers to material of the porous wall, through which the coolant (hydrogen) filters; - and e are for conditions in the porous body, respectively, when y → 0 and y → −∞; g refers to the mixture of gases contained within the porous wall.

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